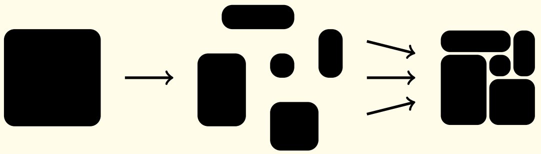
Decomposing Permutation Automata

I. Jecker (University of Warsaw, Poland), N. Mazzocchi (ISTA, Austria) and P. Wolf (University of Bergen, Norway)

Compositionality

instance *factors* *decomposition*



Hardware application : Simplify design

Verification application : Rewrite specification

Formalization for DFAs

- The DFA $\mathcal{B}$ is a factor of $\mathcal{A}$ if:

  $|\mathcal{B}| < |\mathcal{A}| \quad \wedge \quad L(\mathcal{A}) \subseteq L(\mathcal{B})$

- $\mathcal{A}$ is a k-composite if:

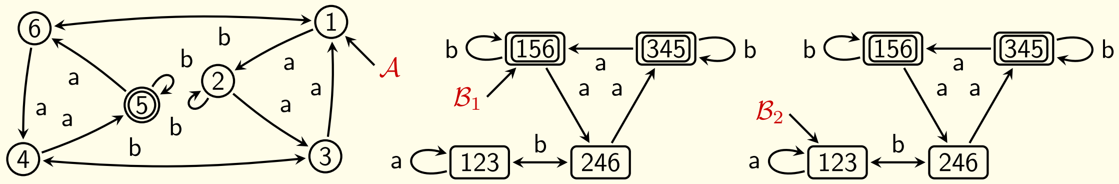
  $L(\mathcal{A}) = \bigcap_{i=1}^{k} L(\mathcal{B}_i)$ where each $\mathcal{B}_i$ is a factor

- $\mathcal{A}$ is composite if it is k-composite for some k, otherwise it is prime.

- The orbit DFA of $\mathcal{A}$ induced by the subset of states S is defined as the DFA obtain by subset contruction on $\mathcal{A}$, where S is the initial state

Key Approach

- A permutation DFA $\mathcal{A}$ is composite iff it can be decomposed into polynomially many orbit DFAs
- If $\mathcal{A}$ is commutative, orbit DFAs defined with a state-space that partitions the state-space of $\mathcal{A}$ suffice



Permutation DFA decomposable into the orbit DFAs induced by $\{1,5,6\}$ and $\{1,2,3\}$: $L(\mathcal{A}) = \bigcap_{i=1}^{2} L(\mathcal{B}_i)$

Summary

Automata class	Composite?	k-Composite?
DFAs	EXPSPACE [1]	PSPACE
Permut. DFAs	NP/FPT*	PSPACE
Commut. permut. DFAs	NLOGSPACE	NP-COMPLETE
Unary DFAs	LOGSPACE [2]	LOGSPACE

* Fixed Parameter Tractable in the number of rejecting states

Large decomposition

For infinitely many $n, m \in \mathbb{N}$, there exist a commutative permutation DFA with n states and m letters, requiring $\left(\sqrt[m]{n} - 1\right)^{m-1}$ factors to be decomposed.

1. **O. Kupferman and J. Mosheiff**
Prime languages
In J. of Inf. and Comput. 2015

2. **I. Jecker, O. Kupferman and N. Mazzocchi**
Unary Prime languages
In MFCS proceedings 2020