CONCUR 2024 – Calgary Canada

Strategic Dominance: A New Preorder for Nondeterministic Processes

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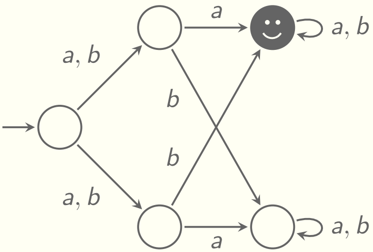
This talk is supported by the ERC-2020-AdG 101020093

Resolving non-determinism



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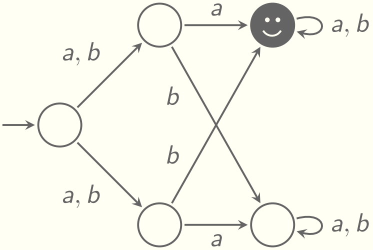

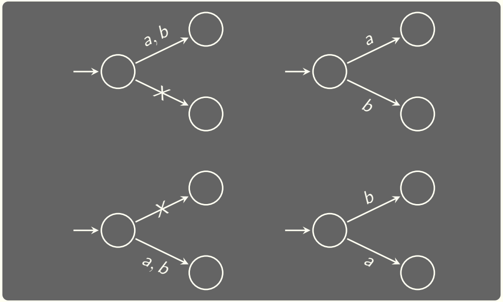

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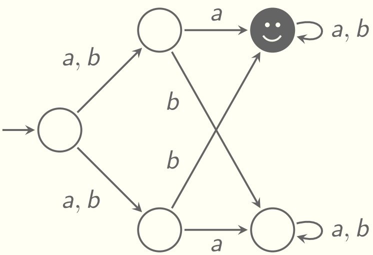

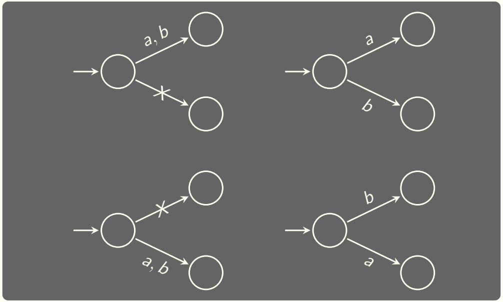

- $\forall w \in \Sigma^\omega : \exists f \in R(\mathcal{A}) :$ ☺ True
- $\exists f \in R(\mathcal{A}) : \forall w \in \Sigma^\omega :$ ☺ False

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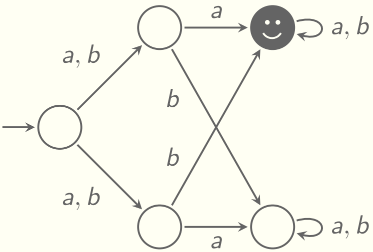

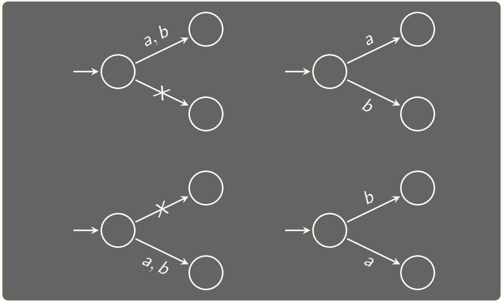

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$\exists f : \forall w :$ ☺ $\implies \forall w : \exists f :$ ☺

Motivating Problems

Trace Inclusion and Simulation

Inclusion of linear-time properties and branching-time properties respectively

History Determinism

Can the non-determinism of a given automaton be expressed by a single resolver?

Inclusion of Hyperproperties

For two set of properties given by non-deterministic automata, is one subset of the another?

Safety

Is the properties given by an automaton safe?

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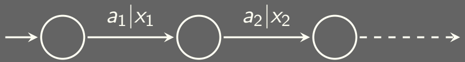
Contribution

One algorithm that solves them all

Automata and Resolvers

Runs



Input: $w = a_1 a_2 \dots$ Output: $x = \mathrm{Val}(x_1 x_2 \dots)$

Value function Val

- Inf (safety)
- Sup (reachability)
- LimInf (co-Büchi)
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Automata and Resolvers

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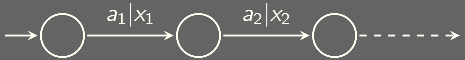

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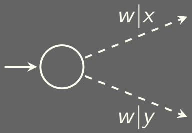

sup $\{$values of runs over $w\}$

Automata and Resolvers

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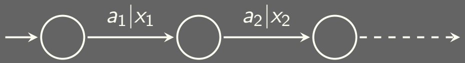

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- $f\colon \mathsf{Edges}^*(\mathcal{A}) \times \Sigma \to \mathsf{State}(\mathcal{A})$
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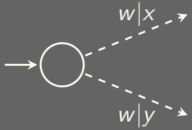

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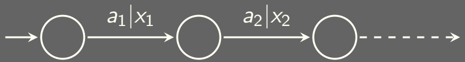

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Trace Inclusion

$$\forall w \in \Sigma^\omega : \mathcal{A}^{\mathsf{sup}}(w) \leq \mathcal{B}^{\mathsf{sup}}(w)$$

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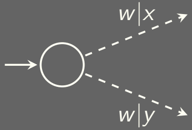

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Trace Inclusion

Resolver Expressiblity

- $\forall w \in \Sigma^\omega : \mathcal{A}^{\mathsf{sup}}(w) \leq \mathcal{B}^{\mathsf{sup}}(w)$ $\qquad \mathcal{A} \subseteq \mathcal{B}$
- $\forall w \in \Sigma^\omega \quad \begin{matrix} \exists f \in R(\mathcal{A}) \colon \forall f' \in R(\mathcal{A}) \colon \mathcal{A}^f(w) \geq \mathcal{A}^{f'}(w) \\ \exists g \in R(\mathcal{B}) \colon \forall g' \in R(\mathcal{B}) \colon \mathcal{B}^g(w) \geq \mathcal{B}^{g'}(w) \end{matrix} \quad \mathcal{A}^f(w) \leq \mathcal{B}^g(w)$

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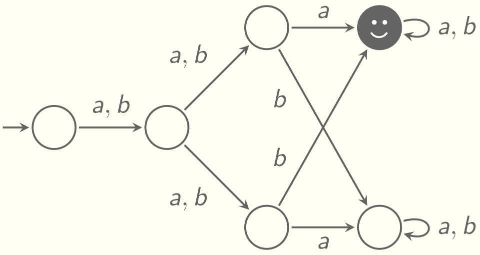
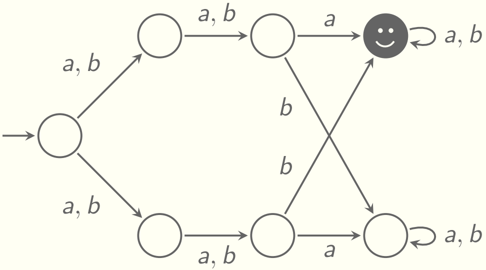
Thm: Deciding Trace Inclusion

$$\begin{aligned} \mathcal{A} \subseteq \mathcal{B} \iff & \forall w \in \Sigma^\omega : \forall f \in R(\mathcal{A}) : \exists g \in R(\mathcal{B}) : \mathcal{A}^f(w) \leq \mathcal{B}^g(w) \\ \iff & \forall w \in \Sigma^\omega : \exists g \in R(\mathcal{B}) : \forall f \in R(\mathcal{A}) : \mathcal{A}^f(w) \leq \mathcal{B}^g(w) \end{aligned}$$

Word Blindness





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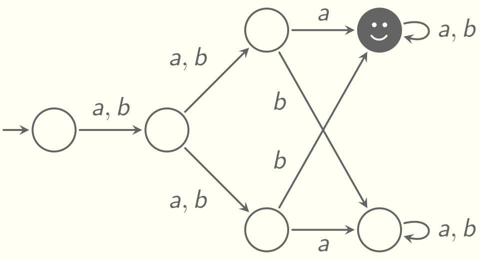

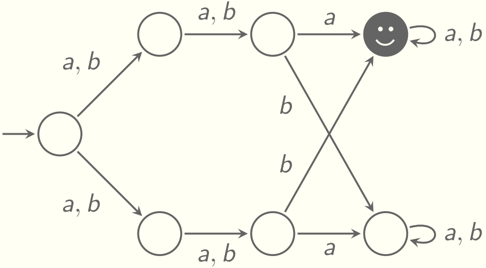

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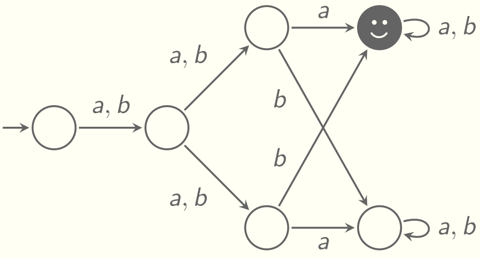

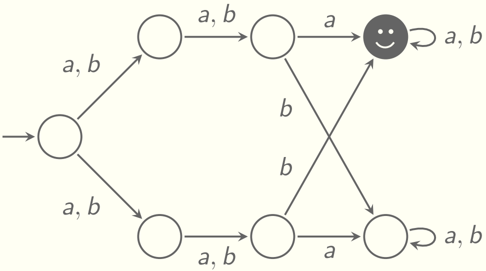

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Thm: $\mathcal{A} \,\underline{\blacktriangleleft}\, \mathcal{A} \iff \mathcal{A}$ is history deterministic

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Simulation

Definition: Quantitative simulation turn-based game $\preceq$

1. a configuration is $(p, q) \in \mathsf{State}(\mathcal{A}) \times \mathsf{State}(\mathcal{B})$, the game starts with initial states
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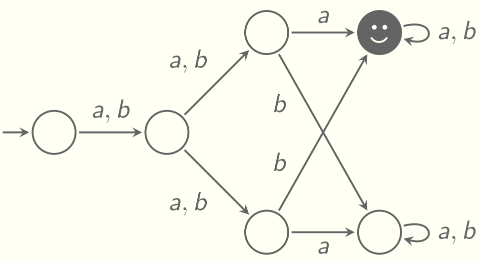

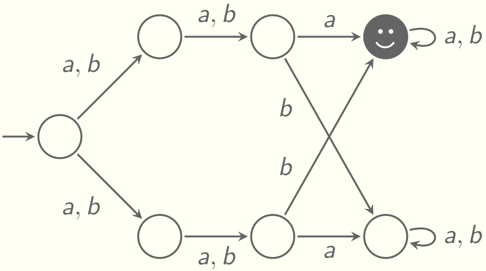

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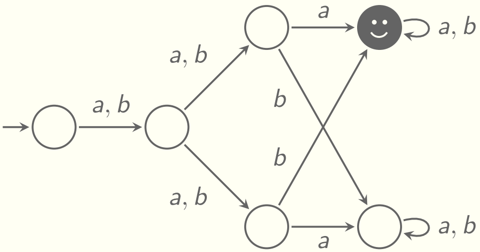

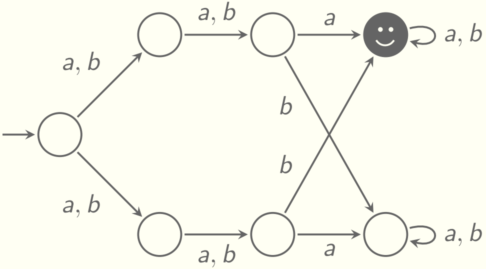

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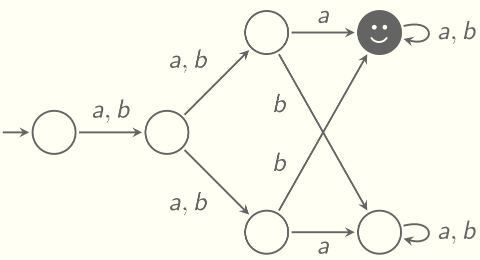
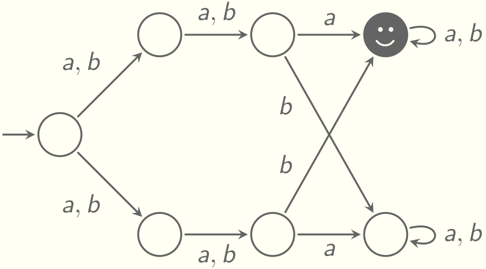
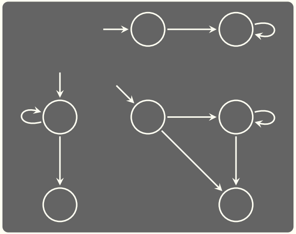
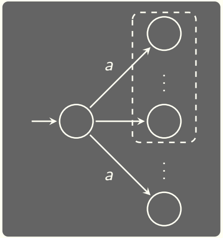




Partial Resolvers and Products





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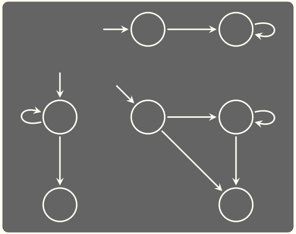

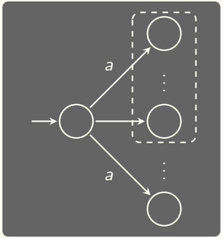

Combining Partial Resolvers

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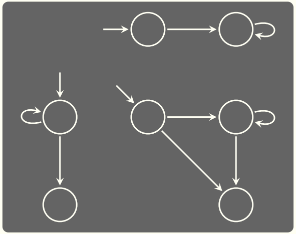

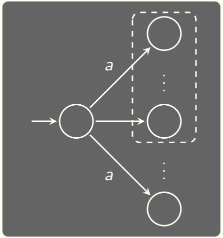

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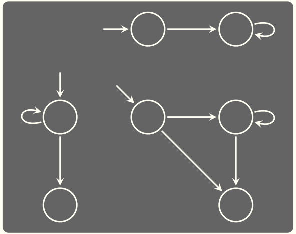

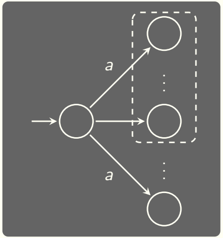

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Thm: $\preceq \iff \trianglelefteq_\times$

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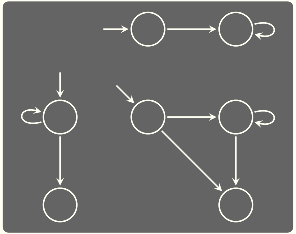

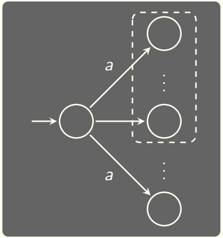

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Thm: $\preceq \iff \trianglelefteq_\times$

$\underline{\blacktriangleleft} \implies \underline{\blacktriangleleft}_\times \implies \preceq \Leftrightarrow \trianglelefteq_\times \implies \trianglelefteq \implies \subseteq$

Hyperproperties and Safety

Definition

- Hyperproperty = Set of properties
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Thm: Deciding Hyper-Inclusion

$$H_{\mathcal{A}} \subseteq H_{\mathcal{B}} \iff \forall f \in R(\mathcal{A}) \colon \exists g \in R(\mathcal{B}) \colon \forall w \in \Sigma^{\omega} \colon \mathcal{A}^f(w) = \mathcal{B}^g(w)$$

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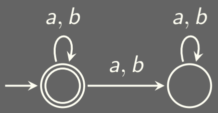
- $H_\mathcal{A} = \{\mathcal{A}^f \mid f \in R(\mathcal{A})\}$
- Incomparable with HyperLTL

Automaton $\mathcal{S}$



Thm: Deciding Hyper-Inclusion

$$H_\mathcal{A} \subseteq H_\mathcal{B} \iff \forall f \in R(\mathcal{A}) \colon \exists g \in R(\mathcal{B}) \colon \forall w \in \Sigma^\omega \colon \mathcal{A}^f(w) = \mathcal{B}^g(w)$$

Hyperproperties and Safety

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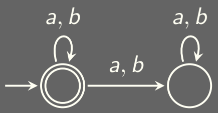

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Thm: Deciding Safety

$$\mathcal{A} \text{ safe} \Leftrightarrow \exists g \in R(\mathcal{S}) \colon \forall w \in \Sigma^\omega \colon \exists f \in R(\mathcal{A}) \colon \forall f' \in R(\mathcal{A}) \colon \mathcal{A}^f(w) \geq \mathcal{A}^{f'}(w) \wedge \mathcal{A}^f(w) = \mathcal{S}^g(w)$$

Hyperproperties and Safety

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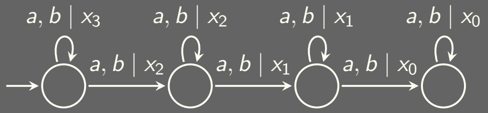

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Resolver Logic

Definition

$$\Psi = \exists w \in \Sigma^\omega : \Psi' \mid \forall w \in \Sigma^\omega : \Psi' \mid \exists f \in R(\mathcal{A}) : \Psi' \mid \forall f \in R(\mathcal{A}) : \Psi' \mid \varphi$$

Word variables	$w \in W$ is interpreted over Σ^ω
Resolver variables	$f \in F_i$ is interpreted over $R(\mathcal{A}_i)$
Integer variables	$x \in X$ is interpreted over $\{\mathcal{A}_i^f(w) \mid f \in F_i, w \in W\}$

- $\mathcal{A}$ range over non-deterministic automata of a finite domain $\{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n\}$
- φ range over existential Presburger formulas, i.e., $\exists FO(\mathbb{Z}, \leq, +, 1)$

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Thm: Model-checking a fixed formula Ψ over $\{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n\}$ is

- d-EXPTIME with $d > 0$ quantifier alternations,
- PTIME with no quantifier alternation

Summary

$$\underline{\blacktriangleleft} \implies \underline{\blacktriangleleft}_{\times} \implies \preceq \iff \trianglelefteq_{\times} \implies \trianglelefteq \implies \subseteq$$

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$$\underline{\blacktriangleleft} \implies \underline{\blacktriangleleft}_\times \implies \underline{\prec} \Leftrightarrow \trianglelefteq_\times \implies \trianglelefteq \implies \subseteq$$

Motivating Problems

- Trace Inclusion
- Strategic Dominance
- Simulation
- History Determinism
- Hyperproperty Inclusion
- Safety
- Emptiness
- Universality

Provided Complexity

- 2 – EXPTIME
- 2 – EXPTIME
- 2 – EXPTIME
- EXPTIME
- 2 – EXPTIME
- 3 – EXPTIME
- PTIME
- 2 – EXPTIME

Best Known

- PSPACE
- PSPACE
- PTIME
- EXPTIME (for LimInf)
- PSPACE
- PSPACE
- PTIME
- PSPACE

Summary

$$\underline{\blacktriangleleft} \implies \underline{\blacktriangleleft}_{\times} \implies \preceq \Leftrightarrow \trianglelefteq_{\times} \implies \trianglelefteq \implies \subseteq$$

Motivating Problems	Provided Complexity	Best Known
Trace Inclusion	2 – ExpTime	PSpace
Strategic Dominance	2 – ExpTime	PSpace
Simulation	2 – ExpTime	PTime
History Determinism	ExpTime	ExpTime (for LimInf)
Hyperproperty Inclusion	2 – ExpTime	PSpace
Safety	3 – ExpTime	PSpace
Emptiness	PTime	PTime
Universality	2 – ExpTime	PSpace

Thank You