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Two-way Parikh automata

Université libre de Bruxelles

FSTTCS 2019 - Bombay

The model in a nutshell

Accepting run

$\sum_{i=1}^{n} \vec{v}_i \models \psi$

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Accepting run of (A, λ, ψ)

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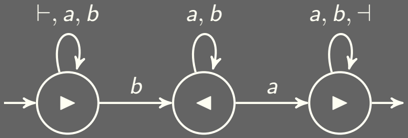
Two-wayness



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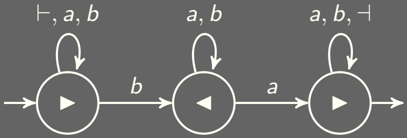


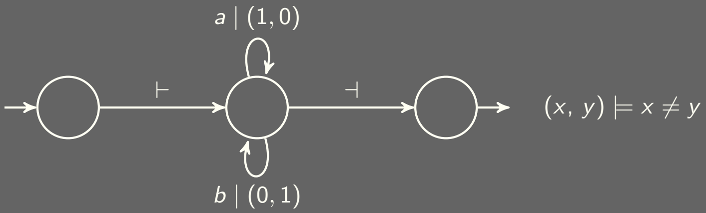
NFA = 2NFA

NPA ≠ 2NPA

One-way

Expressive and decidable formalism

More than NFA $\mathcal{L}_{\neq} = \{u : |u|_a \neq |u|_b\}$



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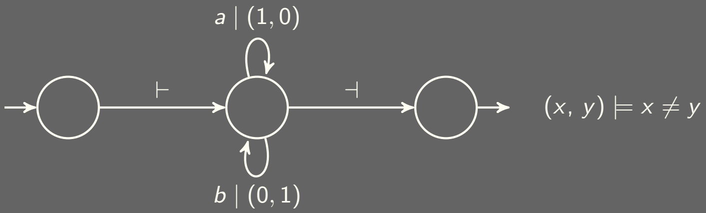

Non-emptiness problem for NPA

- *Decidable [Klaedtke and Rueß, ICALP03]*
- NP-C *with existential formulas [Figueira and Libkin, LICS15]*
- NLogSpace-C *with weak existential formulas [FiliotMR, DLT18]*

Application in transducer theory

Functional transducer equivalence

$$\forall u \in \Sigma^* \quad T_1(u) = T_2(u)$$

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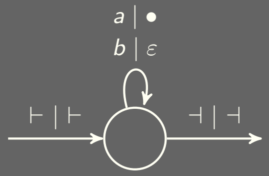

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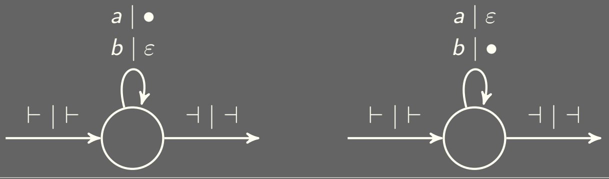

u:	$\vdash$	a	b	a	$\dashv$
$T_1(u)$:	$\vdash$	$\bullet$	ε	$\bullet$	$\dashv$
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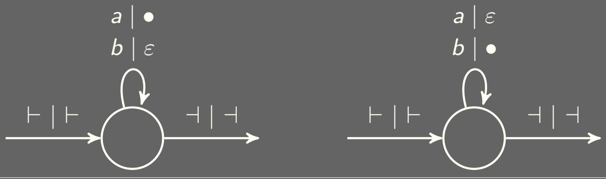

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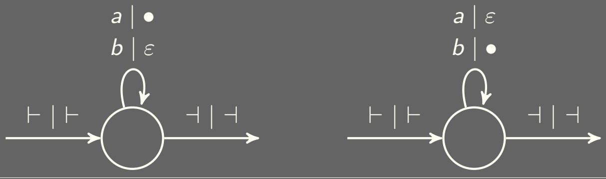

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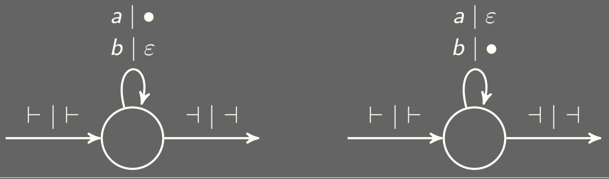

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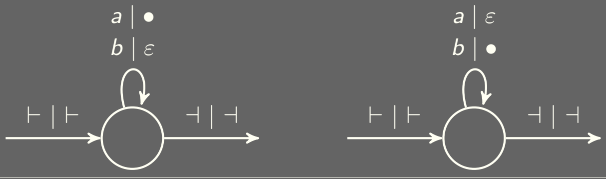

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1. Decidability of non-emptiness
2. Counting positions
3. Recognize non-regular languages

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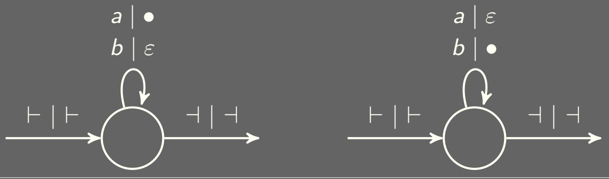

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Let's use NPA!

A powerful tool

Equiv.

1-way functional transducers

A powerful tool

Equiv.	**Non-empt.** NP-C
1-way functional transducers	NPA

1-way functional transducers → NPA: PTime

A powerful tool

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PTime

A powerful tool

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Model-Checking

Pattern Logic [FiliotMR, DLT18]

Pattern logic for transducers

$\mathrm{PL}_{\mathrm{trans}}$ in [Filiot and Mazzocchi and Raskin, DLT18]

$$\varphi ::= (\exists \pi_1 = p_1 \xrightarrow{u_1 | v_1} q_1), \ldots, (\exists \pi_n = p_n \xrightarrow{u_n | v_n} q_n)\ \mathcal{C}$$

$$\mathcal{C} ::= \mathcal{C} \vee \mathcal{C} \mid \mathcal{C} \wedge \mathcal{C} \mid P \mid \neg P \mid P_{\mathrm{out}}$$

P	$u_1 \sqsubseteq u_2$
	$\mathtt{init}(q) \mid \mathtt{final}(q)$
$\mathbf{P}_{\mathrm{out}}$	$v_1 \neq v_2$

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Functionality

$$\exists \pi_1 = p_1 \xrightarrow{u_1 \mid v_1} q_1, \exists \pi_2 = p_2 \xrightarrow{u_2 \mid v_2} q_1 \ \bigwedge \begin{cases} u_1 = u_2 \wedge v_1 \neq v_2 \\ \wedge_{i=1}^{2}\ \mathtt{init}(p_i) \wedge \mathtt{final}(q_i) \end{cases}$$

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P	$u_1 \sqsubseteq u_2 \mid u \in L \mid \lvert u_1 \rvert \leq \lvert u_2 \rvert$
	$\texttt{init}(q) \mid \texttt{final}(q) \mid \pi_1 = \pi_2 \mid q_1 = q_2$
$\mathbf{P_{out}}$	$t_1 \not\sqsubseteq t_2 \mid \lvert t_1 \rvert <_{len} \lvert t_2 \rvert \mid \lvert t_1 \leq_{len} \lvert t_2 \rvert \mid t \in N \mid t \notin N$

- L, N range over regular language represented as an NFA
- $t, t' \in \mathit{Terms}(\{v_1, \ldots, v_n\}, \cdot, \varepsilon)$

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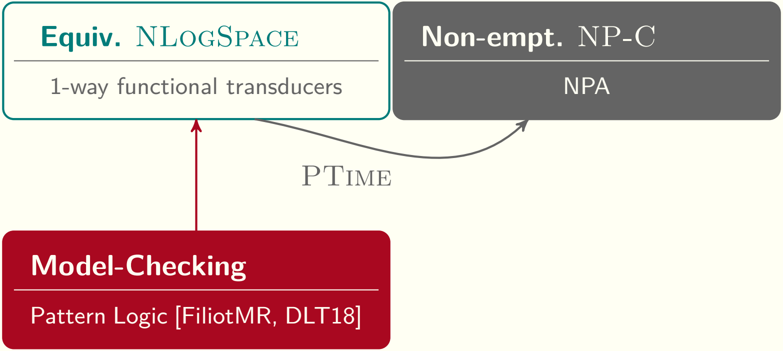

Two-way

A powerful tool

Equiv. NLogSpace

1-way functional transducers

Non-empt. NP-C

NPA

PTime

Model-Checking

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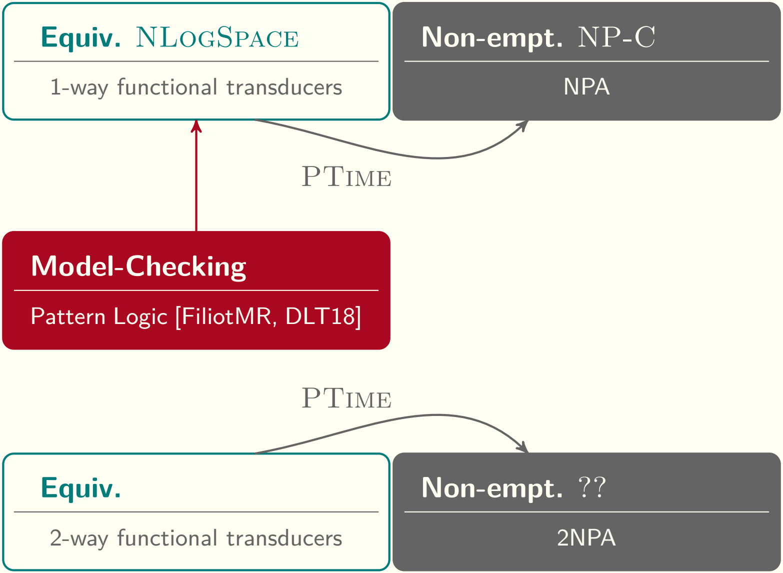
Equiv.

2-way functional transducers

A powerful tool



How two-wayness allows $\times$

From Hilbert's 10th problem

- *Non-emptiness for 2NPA is undecidable*

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Figure: A 2NPA recognising $\{a^n \diamond a^m \diamond a^{n \times m} \mid n, m \in \mathbb{N}\}$

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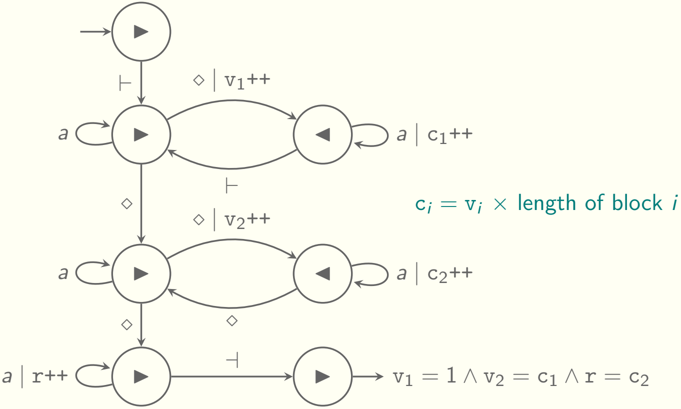

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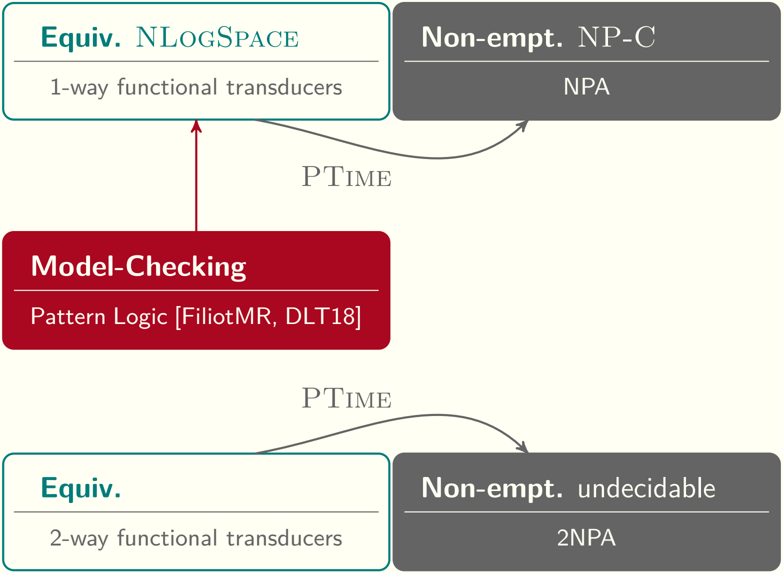

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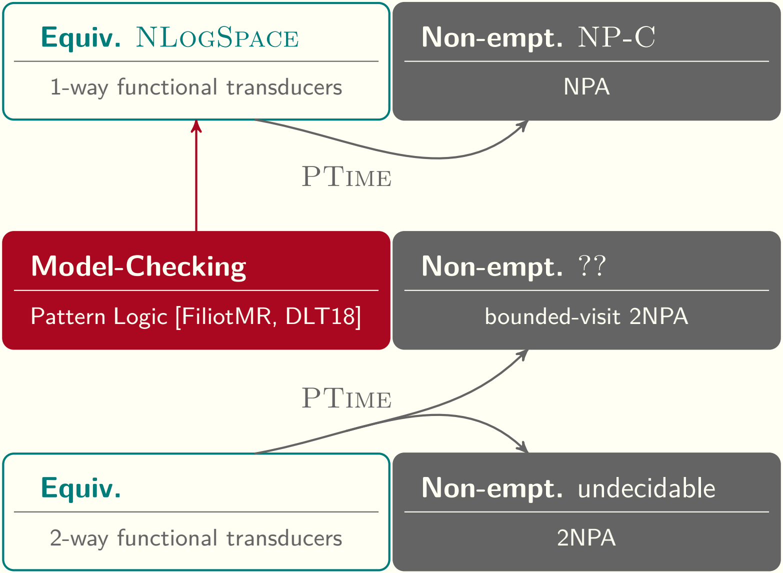

How two-wayness can be removed

From Aho and Hopcroft and Ullman construction

- *bounded-visit 2NPA to NPA is in* EXPTIME
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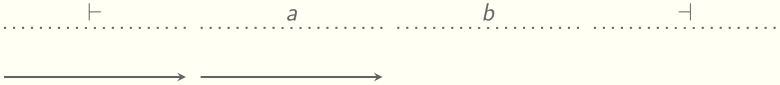

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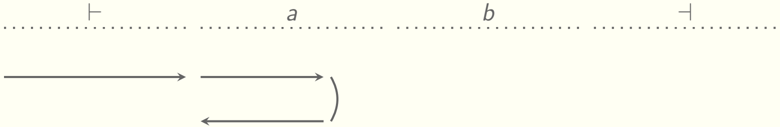

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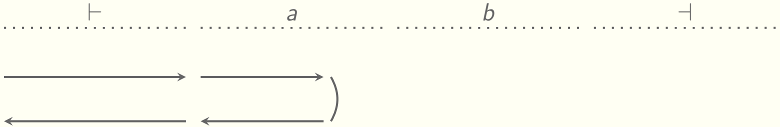

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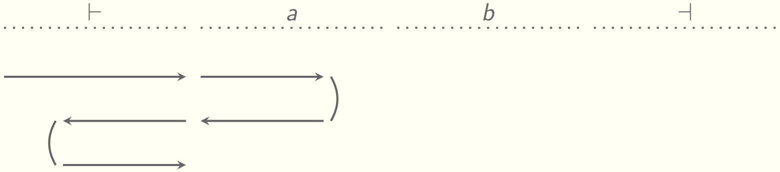

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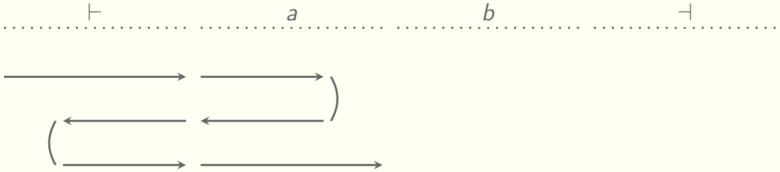

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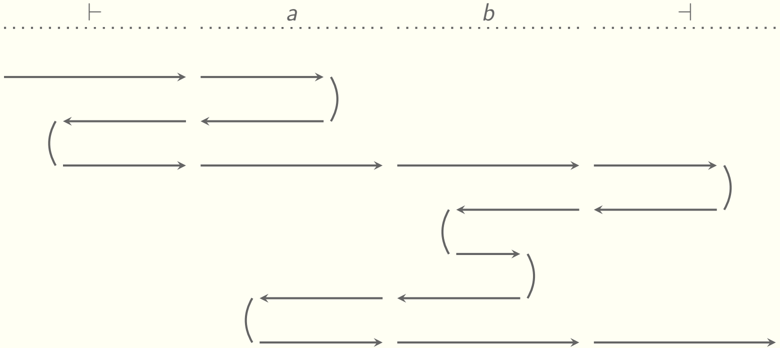

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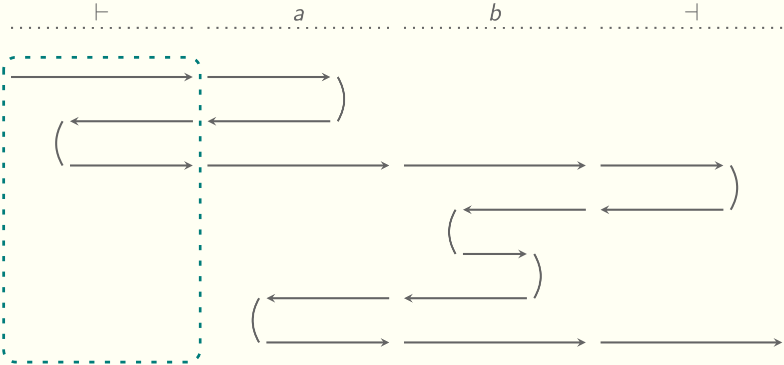

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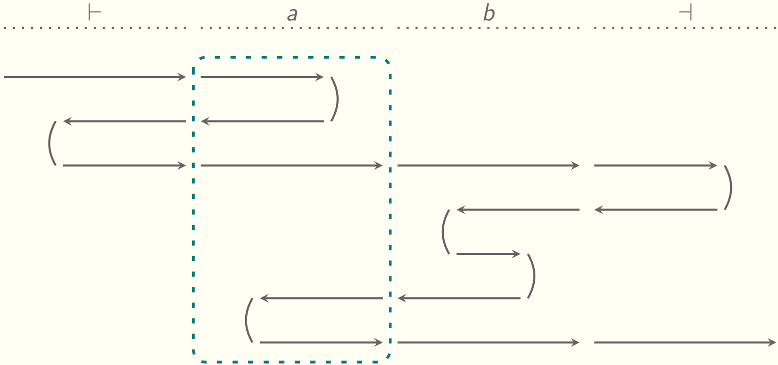

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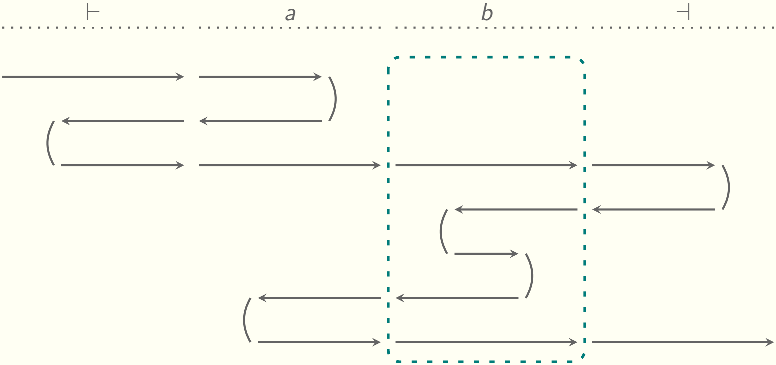

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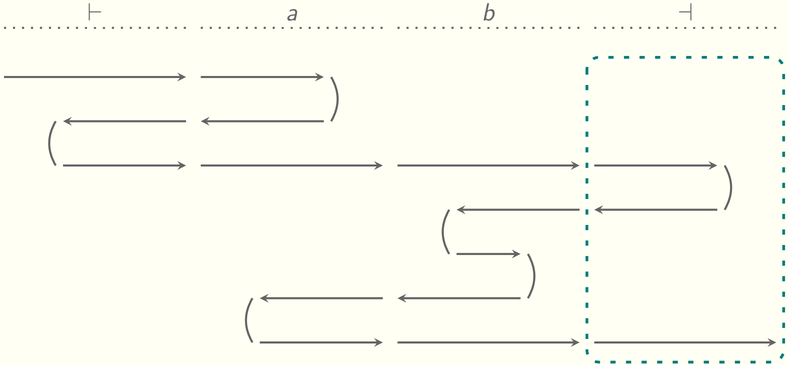

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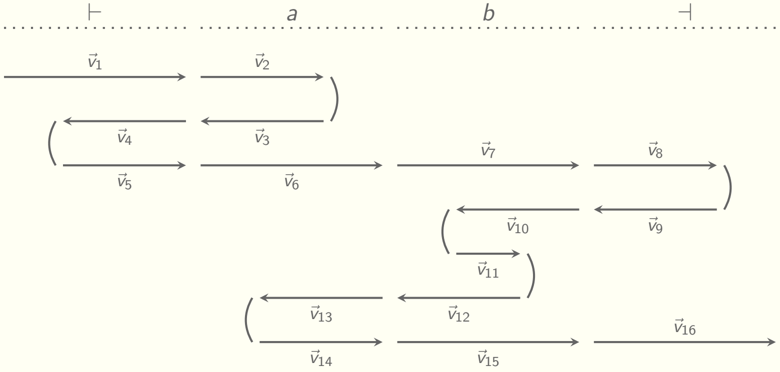
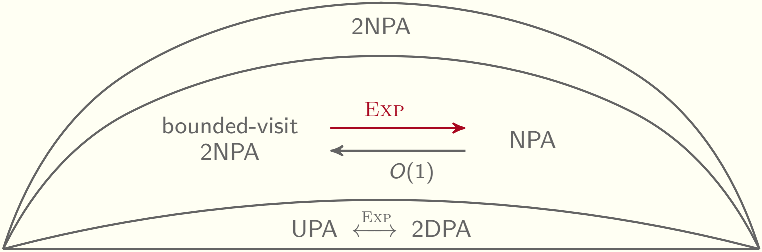


Comparing expressive power



Remark

UPA to bounded-visit 2DPA is in ExpTime [DartoisFJL, ICALP17]

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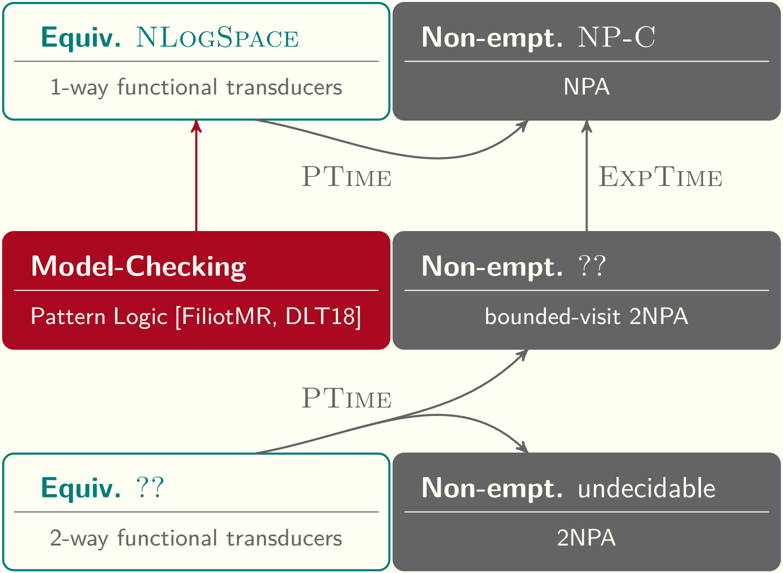

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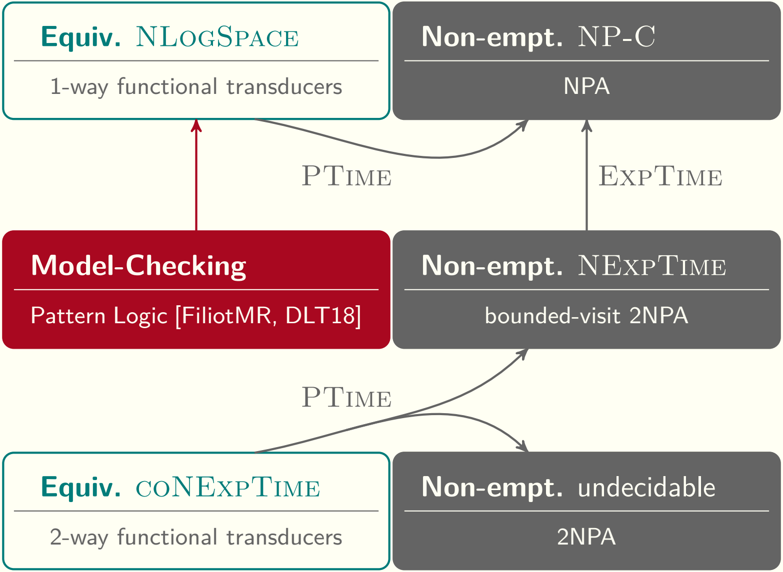

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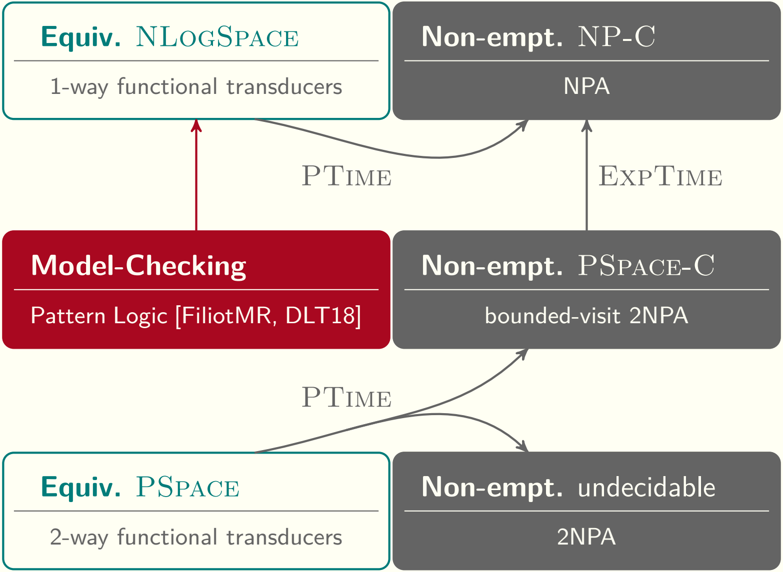
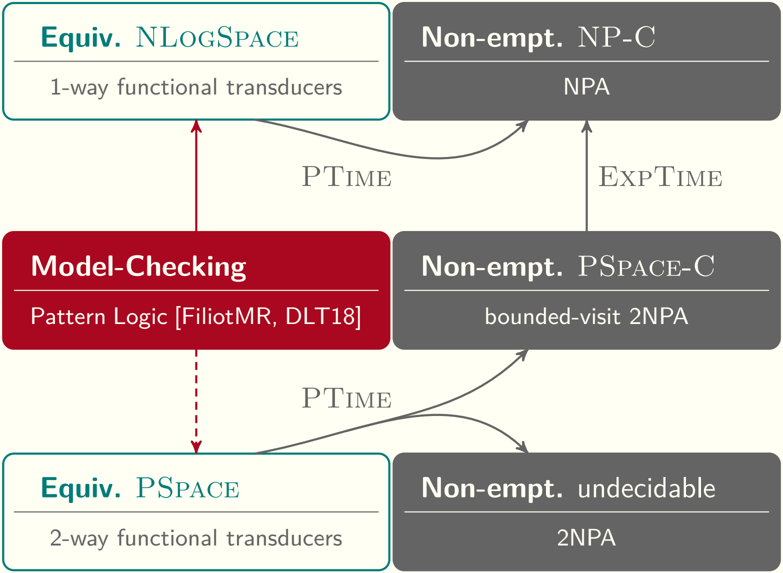


A powerful tool



Generalised constraints

Universality ... and beyond

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$$\Sigma^{\text{P}}_{i+1} \overset{\text{def}}{=} \text{NP}^{\Sigma^{\text{P}}_i} \qquad \Sigma^{\text{Exp}}_{i+1} \overset{\text{def}}{=} \text{NExpTime}^{\Sigma^{\text{P}}_i}$$

$$\Pi^{\text{P}}_{i+1} \overset{\text{def}}{=} \text{coNP}^{\Sigma^{\text{P}}_i} \qquad \Pi^{\text{Exp}}_{i+1} \overset{\text{def}}{=} \text{coNExpTime}^{\Sigma^{\text{P}}_i}$$

It's time to conclude

<table>
<tr><th></th><th>Automata</th><th>Non-emptiness</th><th>Universality</th></tr>
<tr><td></td><td>2NPA</td><td>undecidable</td><td rowspan="2">undecidable</td></tr>
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</table>

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Question?