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Highlights 2019 - Warsaw

Two-way Parikh automata: tool in transducer theory

Definitions

Presburger acceptance

$\sum_{i=1}^{n} \vec{v_i} \models \psi$

Presburger formulas ($\exists$FO$[\mathbb{Z}, \leq, +]$)

$$\psi := \neg\psi \mid \exists x\ \psi \mid \psi \wedge \psi \mid \psi \vee \psi \mid t \leq t$$

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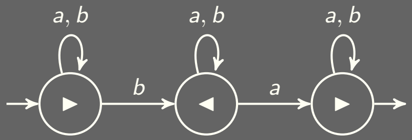
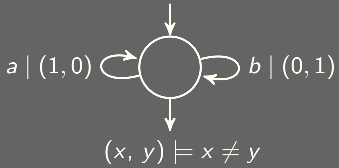
2-wayness



NFA = 2NFA

Expressive and decidable formalism

More than NFA $\mathcal{L}_{\neq} = \{u : |u|_a \neq |u|_b\}$



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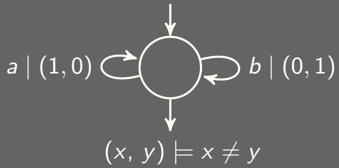

Non-emptiness problem for NPA

- *Decidable [Klaedtke and Rueß, ICALP03]*
- NP-C *with existential formulas [Figueira and Libkin, LICS15]*

Application in transducer theory

Functional transducer equivalence

$$\forall u \in \Sigma^* \quad T_1(u) = T_2(u)$$

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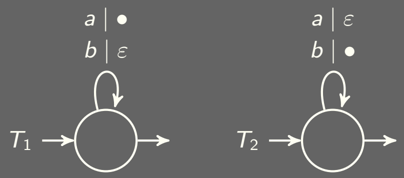

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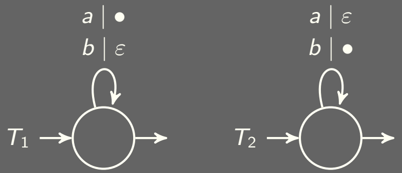

u:	a	b	a
$T_1(u)$:	$\bullet$	ε	$\bullet$
$T_2(u)$:	ε	$\bullet$	ε

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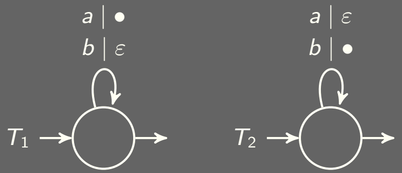

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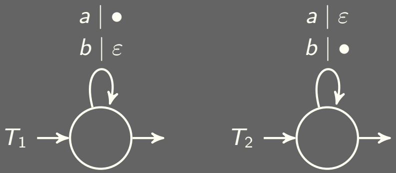

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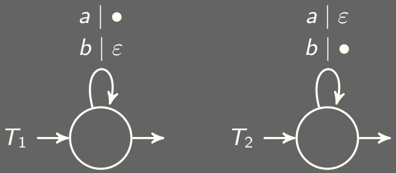

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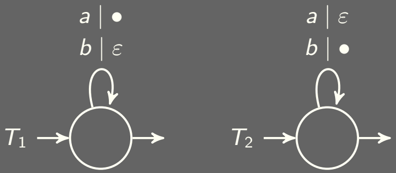

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2. Counting positions
3. Recognize non-regular languages

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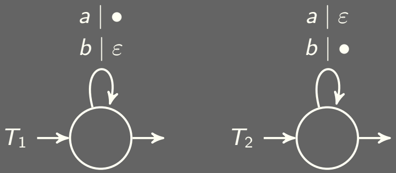

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Let's use NPA!

A powerful tool

Equiv. ??

1-way functional transducers

A powerful tool

Equiv. coNP

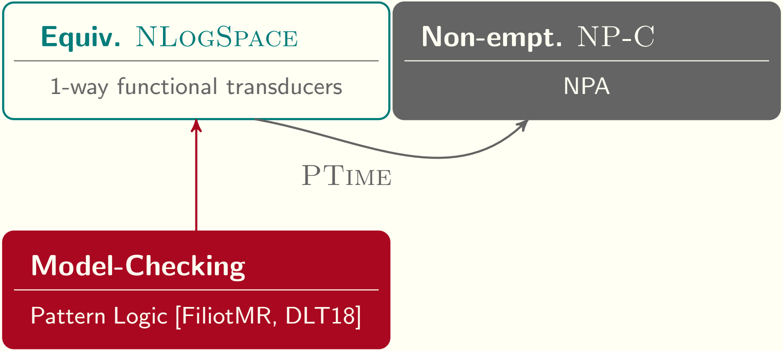
1-way functional transducers

Non-empt. NP-C

NPA

PTime

A powerful tool



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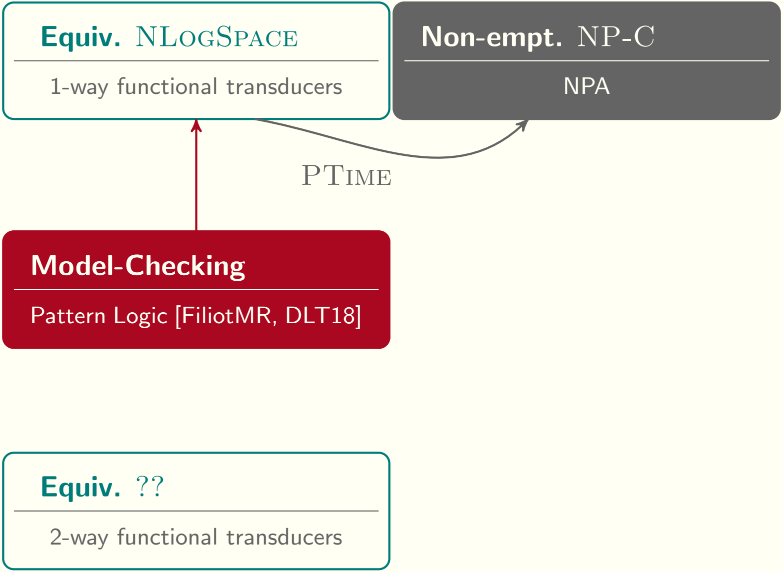

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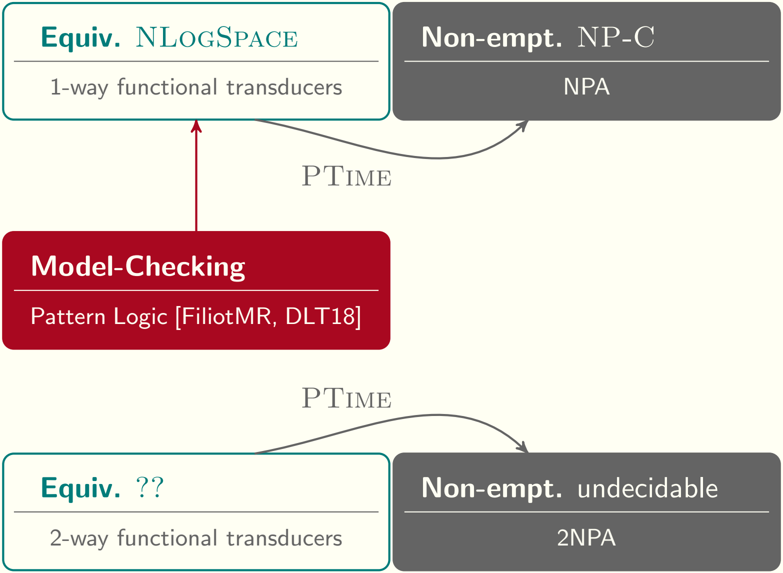

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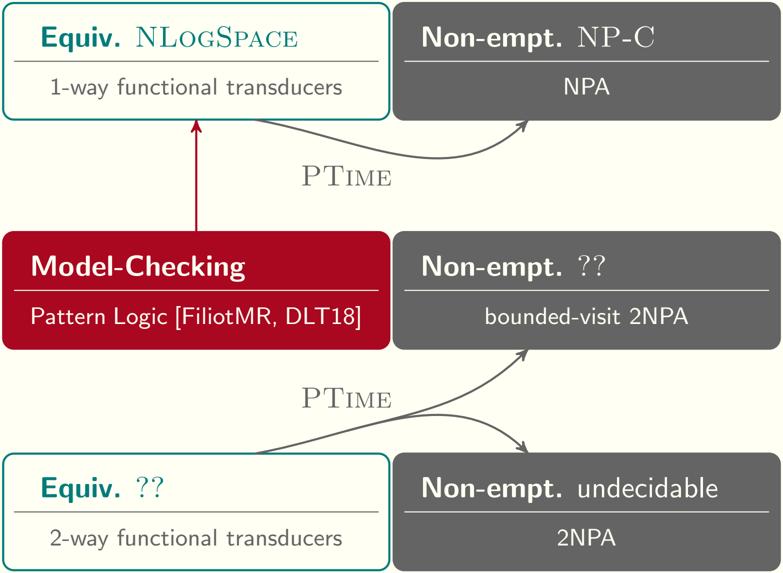

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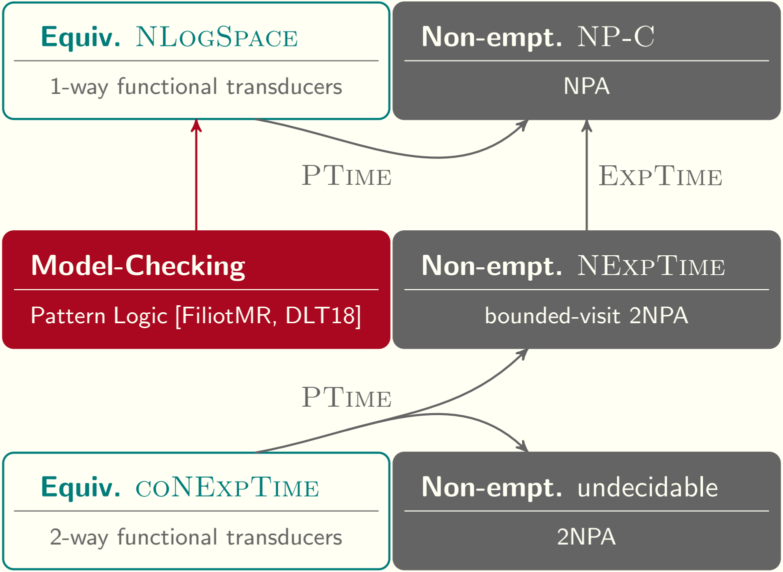

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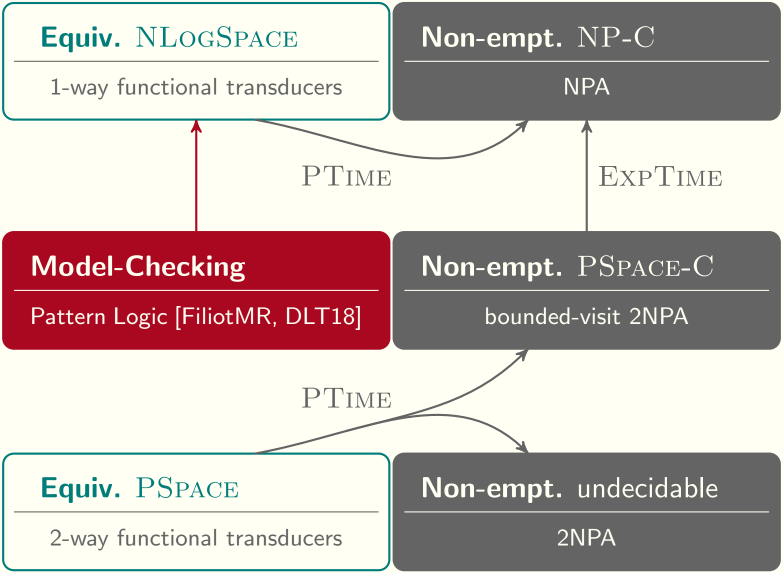

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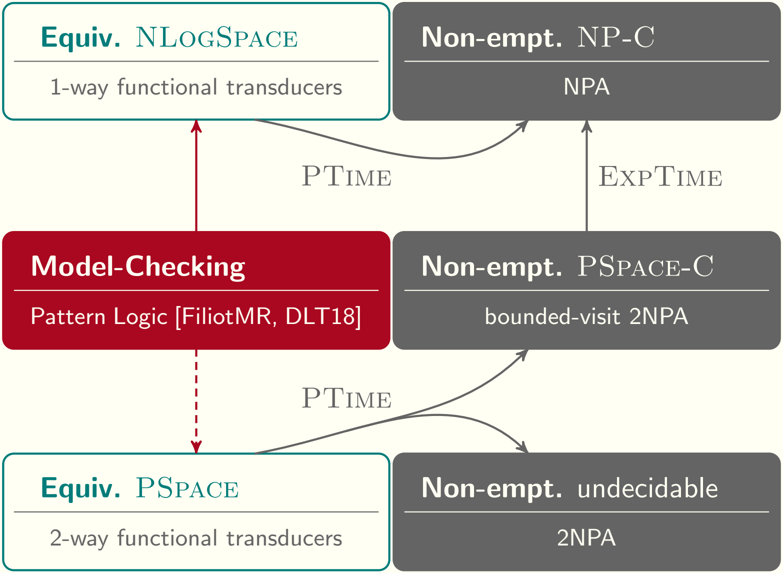

Complexities with two-wayness

	Automata	Non-emptiness	Universality
	2NPA	undecidable	undecidable
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[FiliotGM, FSTTCS19]
thanks to [Haase, LICS14]

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Question?